Metodi di Approssimazione

- MSc course (Laurea magistrale); 42 hours, 6 credits.
- 2nd semester (Spring).

What is this course about? Not approximation theory, mostly. (!)

Selected topics in advanced linear algebra, close to (some) industry applications and (some) modern research themes.

Themes

- Methods to compute matrix functions;
- Methods to solve some specific matrix equations;
- Applications to control theory.

Movie trailer 1: matrix functions

Given a scalar function $f : U \subseteq \mathbb{C} \to \mathbb{C}$, can we extend it to $A \in \mathbb{C}^{n\times n}$?
You already know $\exp(A) = I + A + \frac{A^2}{2} + \ldots$

Idea: Either via series expansion, or eigendecomposition.

Higher derivatives of f pop up unexpectedly:

$$f\left(\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} f(0) & f'(0) & \frac{1}{2}f''(0) \\ 0 & f(0) & f'(0) \\ 0 & 0 & f(0) \end{bmatrix}.$$

Techniques to compute them involve:

- matrix decompositions;
- some approximation theory (replace f with a polynomial or rational function);
- Iterations, e.g., $X_{k+1} = \frac{1}{2}(X_k + X_k^{-1})$;
- ad-hoc tricks such as $\exp(2A) = \exp(A)^2$.

Movie trailer 2: matrix equations

Algebraic Riccati equations

Find $X \in \mathbb{R}^{n \times n}$ that solves

$$XCX - AX + XD - B = 0.$$

This appears in several applications, e.g., control theory.
How to solve it?

(Block) eigenvalue problem in disguise: find X, $\Lambda = CX + D$ s.t.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} X \\ I \end{bmatrix} = \begin{bmatrix} X \\ I \end{bmatrix} \Lambda.$$

Solution (in the generic case): take n of the $2n$ eigenvectors of that block matrix, and choose within their span

$$\begin{bmatrix} X \\ I \end{bmatrix} = \begin{bmatrix} v_1 & v_2 & \dots & v_n \end{bmatrix} W.$$

Info

Prereqs

- Numerical analysis
- Scientific computing

Synergy with other courses from the same area, e.g., numerical methods for Markov chains.

Not a 'generalist' course on numerical analysis / linear algebra.

Course format

- Frontal lectures with Matlab examples.
- Tablet notes + slides available.

Studying

Books

- Higham *Functions of Matrices*.
- Datta, *Numerical Methods for Linear Control Theory*.

Exam

Presentation on a research paper (theory + implementing numerical examples).