What is this course about

Not approximation theory, mostly. (!)

A few selected advanced topics in linear algebra, close to (some of) the themes of our research group in Pisa.

Themes

- Methods to compute matrix functions;
- Methods to solve some specific matrix equations;
- Applications to control theory.

Movie trailer 1: matrix functions

How to define $f(A)$ for an analytic function f? You have already seen $\exp(A) = I + A + \frac{A^2}{2} + \dots$

Either via a series expansion or as

$$f(A) = f(V \Lambda V^{-1}) = V \operatorname{diag}(f(\lambda_1), f(\lambda_2), \dots, f(\lambda_m)) V^{-1}.$$

Higher derivatives of f pop up unexpectedly:

$$f\left(\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} f(0) & f'(0) & \frac{1}{2}f''(0) \\ 0 & f(0) & f'(0) \\ 0 & 0 & f(0) \end{bmatrix}.$$

Techniques to compute them involve matrix decompositions, some approximation theory (replace f with a polynomial or rational function), Cauchy integrals, and some ad-hoc tricks such as $\exp(2A) = \exp(A)^2$.

Movie trailer 2: matrix equations

Algebraic Riccati equations

Find $X \in \mathbb{R}^{n\times n}$ that solves

$$XCX - AX + XD - B = 0.$$

Appears in several applications, e.g., control theory. How to solve it?

(Block) eigenvalue problem in disguise: find X, $\Lambda = CX + D$ s.t.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} X \\ I \end{bmatrix} = \begin{bmatrix} X \\ I \end{bmatrix} \Lambda.$$

Solution (in the generic case): take n of the $2n$ eigenvalues of that block matrix, and choose within their span

$$\begin{bmatrix} X \\ I \end{bmatrix} = \begin{bmatrix} v_1 & v_2 & \dots & v_n \end{bmatrix} W.$$

Movie trailer: matrix sign

Newton for the matrix sign

$A_{k+1} = \frac{1}{2}(A_k + A_k^{-1}), \qquad A_0 = A.$

If

$$A_k = V \operatorname{diag}(\lambda_1, \lambda_2, \dots, \lambda_m) V^{-1},$$

then

$$A_{k+1} = V \operatorname{diag}\left(\frac{\lambda_1 + \lambda_1^{-1}}{2}, \frac{\lambda_2 + \lambda_2^{-1}}{2}, \dots, \frac{\lambda_m + \lambda_m^{-1}}{2}\right) V^{-1},$$

Each eigenvalue evolves independently, converging to one of the two fixed points ± 1.

Splits the spectrum of A in two: $\ker(A_\infty - I)$ and $\ker(A_\infty + I)$.

- Can be used to find eigenvalues recursively, as a "matrix product-heavy" algorithm...
- ...and to solve algebraic Riccati equations.

Course features

Prereqs

- Numerical analysis
- Scientific computing

Synergy with other courses from the same area, e.g., numerical methods for Markov chains.

Not a 'general' course on numerical analysis / linear algebra.

Course format

- Frontal lectures with various Matlab examples.
- Tablet notes produced during the lectures + slides available for later.
- Possibly (if manageable) each student will give a short mini-lecture on a specific topic during the course.

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Studying

Books

- Higham *Functions of Matrices*.
- Datta, *Numerical Methods for Linear Control Theory*.

Exam

Seminar on a research paper on these topics (typically showing some theory + implementing numerical examples).

Details

- 42 hours, 6 credits.
- 2nd semester (Spring).

I hope to see you in a few months!

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